Possible Sources of Instabilities in the Cochlea

Spontaneous Otoacoustic Emissions

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Figure courtesy J. Meiss and D. Simpson, DSWeb media gallery.

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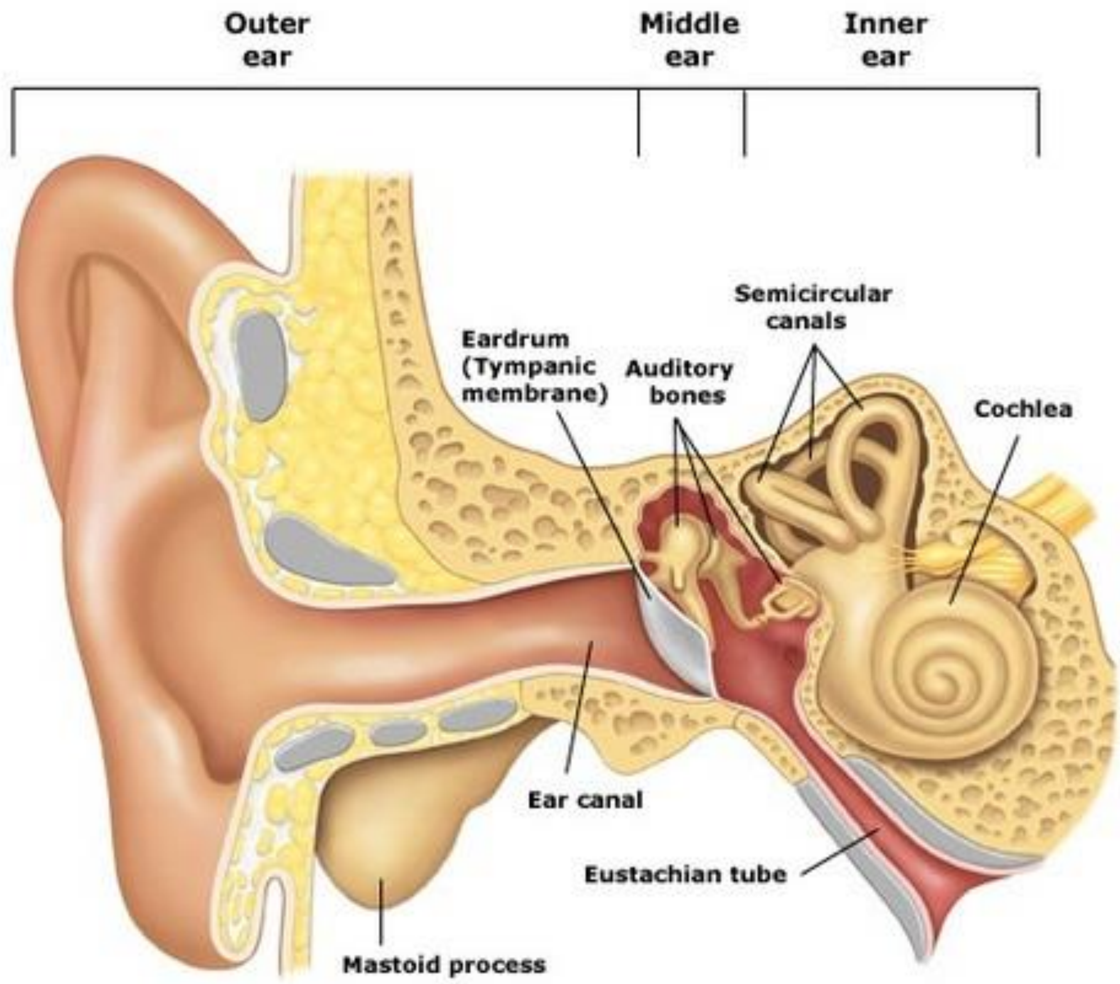
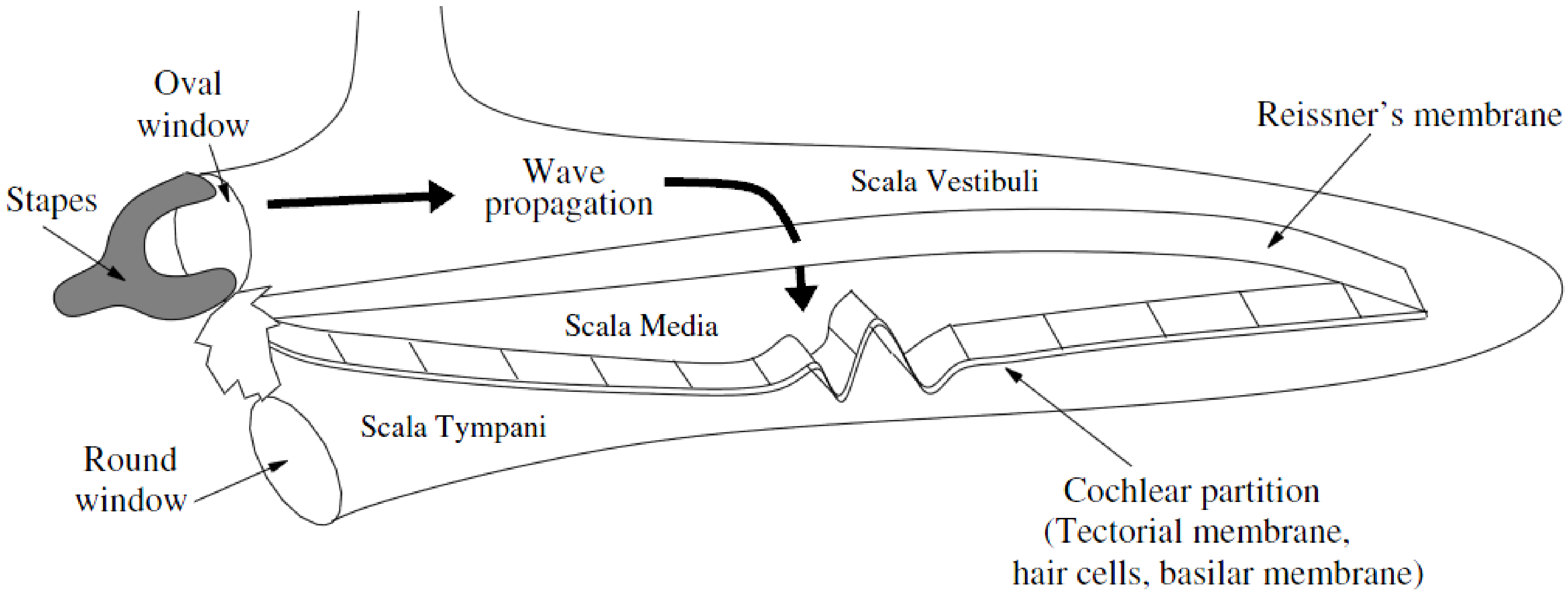
Outline

- Brief Physiology
- Features of Cochlear Response
- Mathematical Model
- Simulations and Dynamic Mode Decomposition
- Instabilities in the Model
- Conclusion

Brief Physiology of the Ear

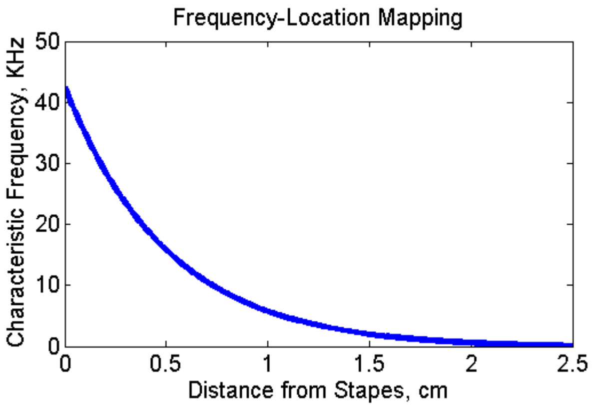


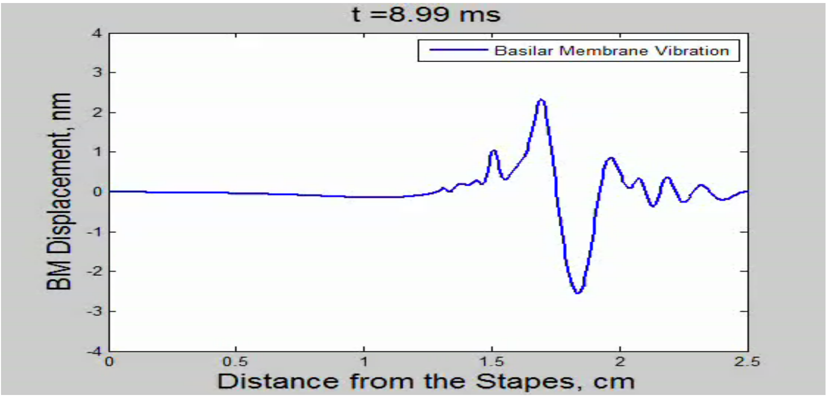
The Cochlea


Oval
window
Stapes
Wave
propagation
Scala Vestibuli
Reissner's membrane
Scala Media
Scala Tympani
Round
window
Cochlear partition
(Tectorial membrane,
hair cells, basilar membrane)

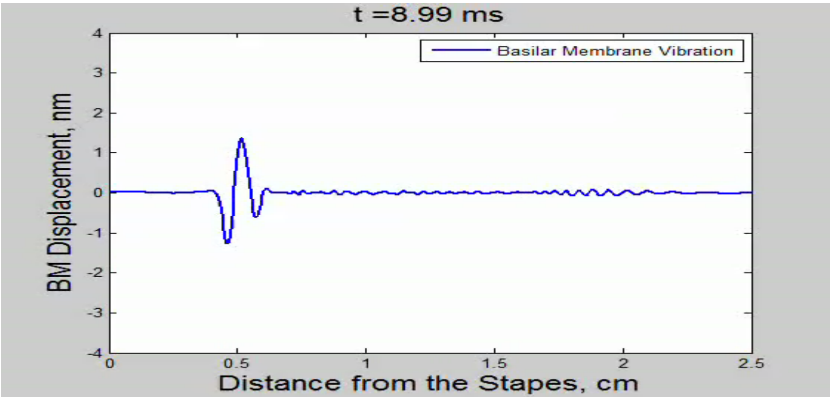
Features of Cochlear Response

- Frequency-Spatial Correlation





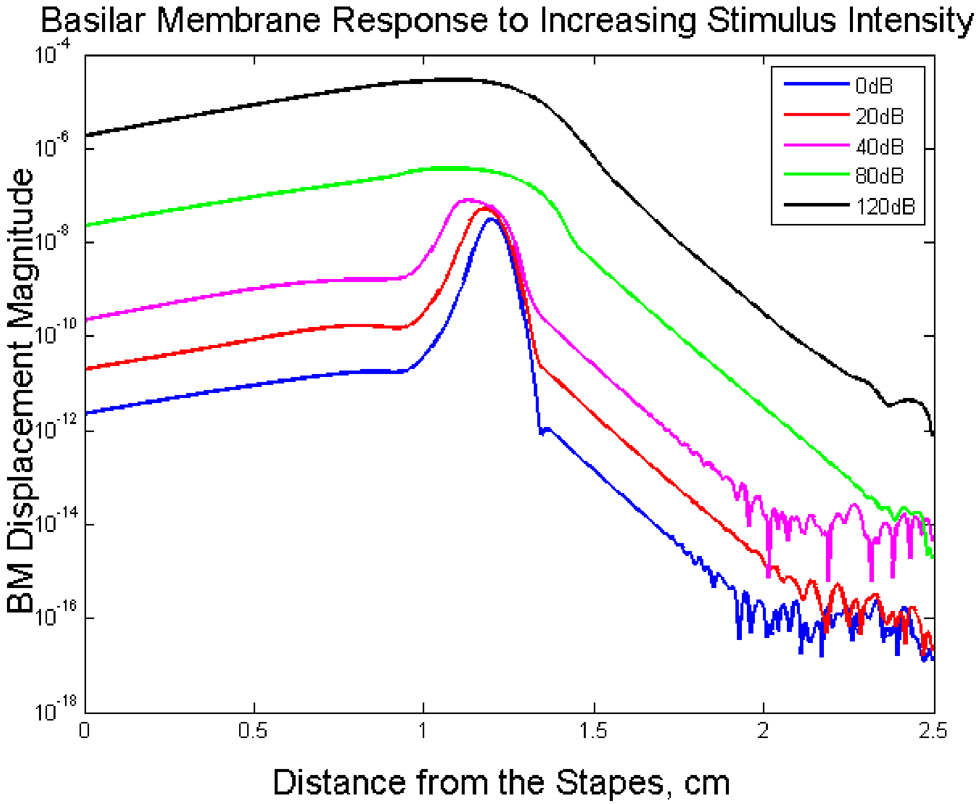
Response to Low Frequency



Response to High Frequency

Features of Cochlear Response

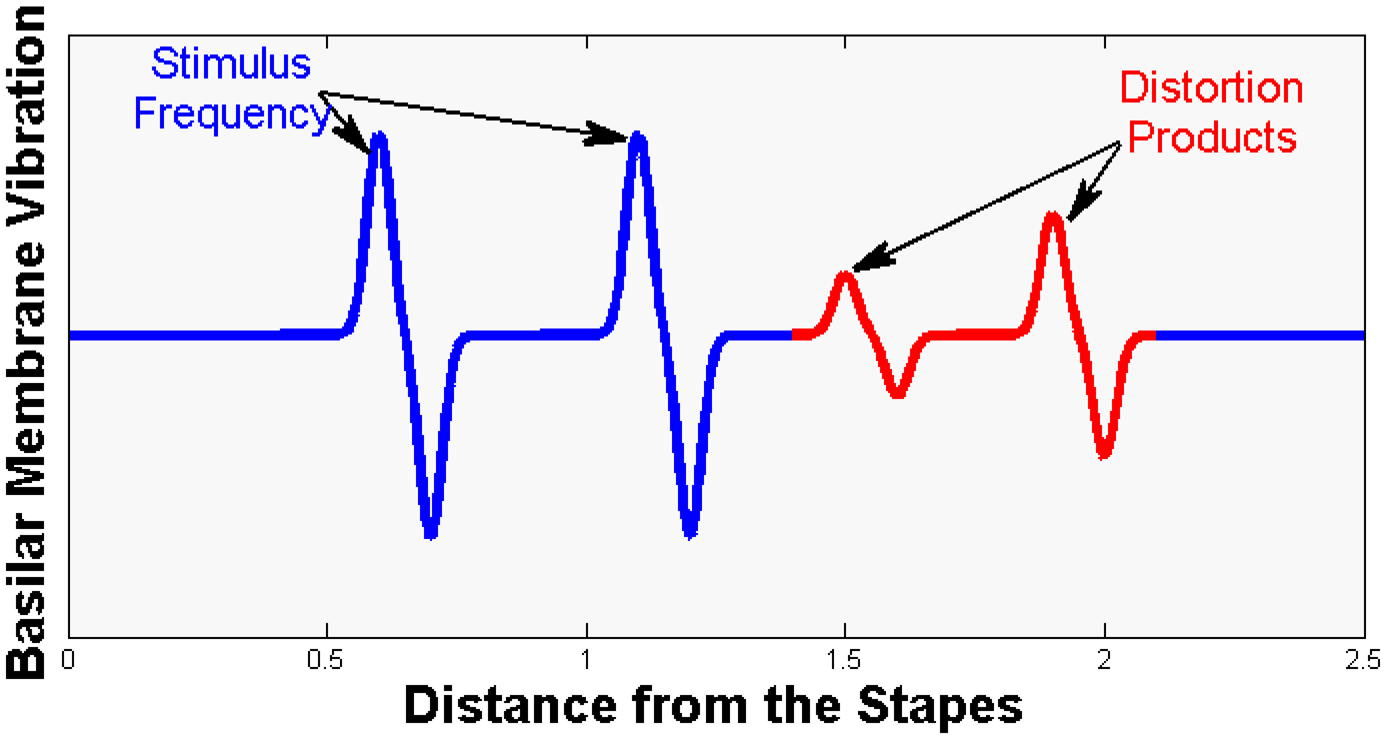
- Frequency-Spatial Correlation
- Very Wide Dynamic Range



Dynamic Range: 0dB to 120dB

Features of Cochlear Response

- Frequency-Spatial Correlation
- Wide Dynamic Range
- Distortion Products



Features of Cochlear Response

- Frequency-Spatial Correlation
- Wide Dynamic Range
- Distortion Products
- Spontaneous Otoacoustic Emissions

Mathematical Model

$p(x,y,t)$: Fluid Pressure

$u(x,t)$: Basilar Membrane Displacement

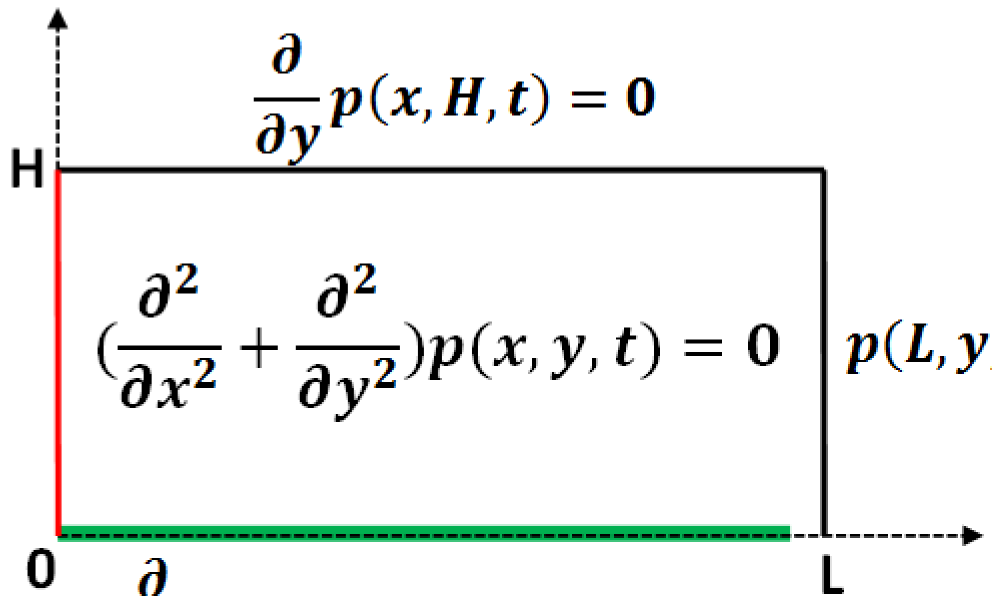
$s(t)$: Stapes Displacement

$$\frac{\partial}{\partial y} p(x,H,t) = 0$$

$$\frac{\partial}{\partial x} p(0,y,t) = 2\rho\, \ddot{s}(t)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) p(x,y,t) = 0$$

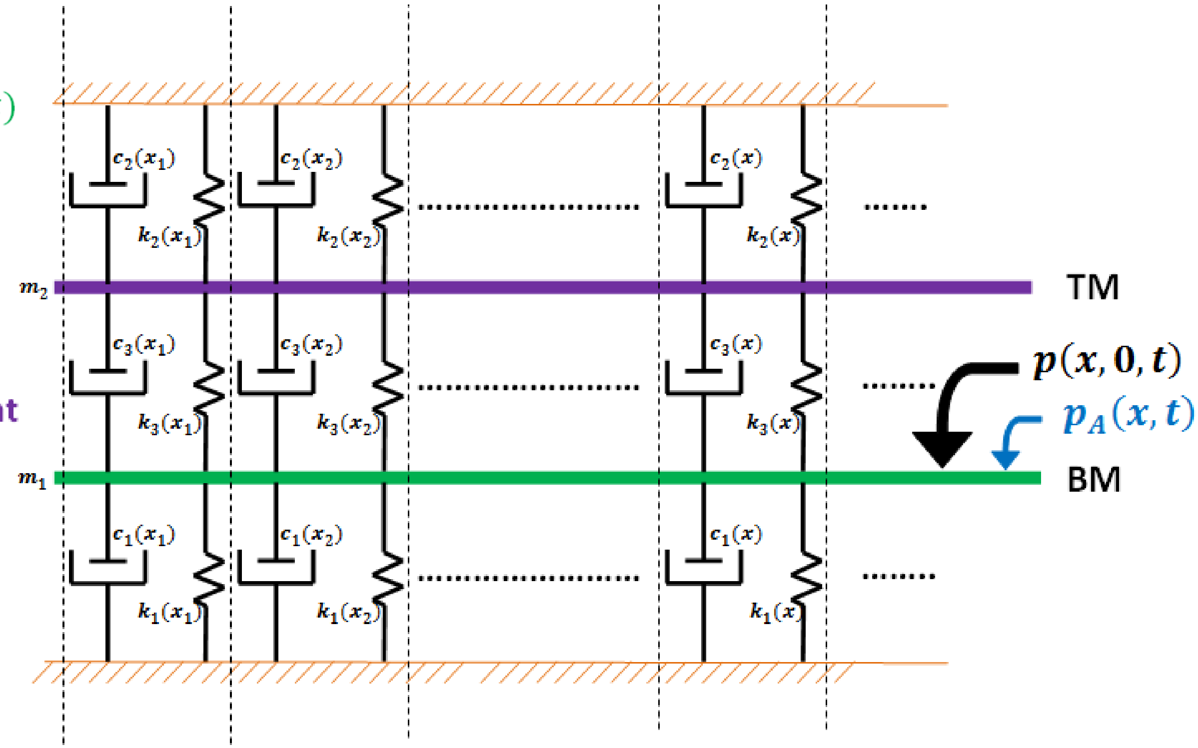
$$p(L,y,t) = 0$$



$$\frac{\partial}{\partial y} p(x,0,t) = 2\rho\, \ddot{u}(x,t)$$

$$p(x,0,t) = \mathcal{L}_1(\ddot{s}) + \mathcal{L}_2(\ddot{u})$$

Mathematical Model



$\gamma(u)$: Active Gain

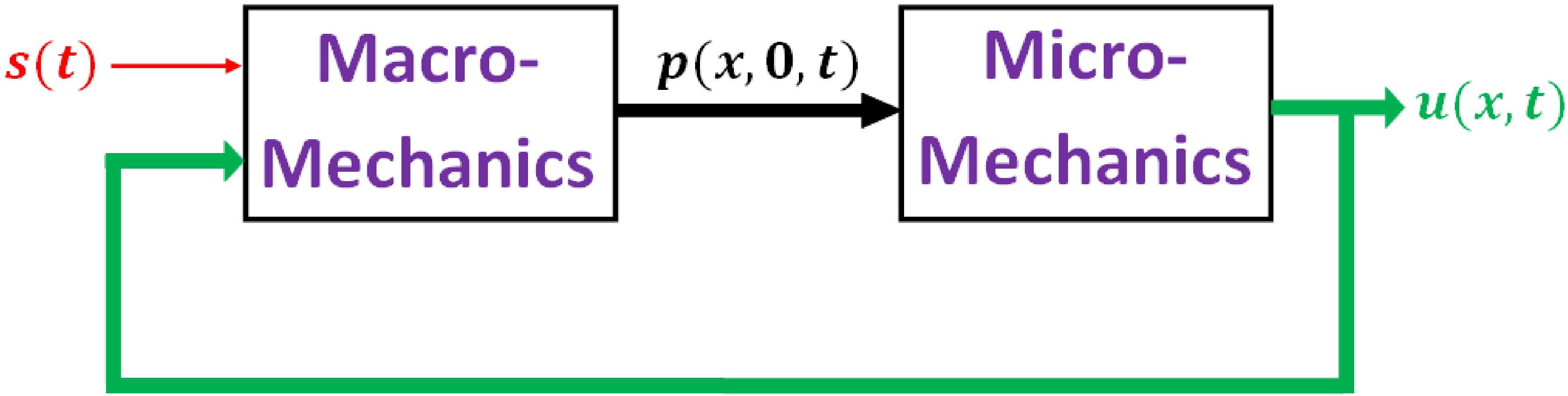
$v(x,t)$: Tectorial Membrane Displacement

$$[M_p]\begin{bmatrix}\ddot{u}(x,t)\\ \ddot{v}(x,t)\end{bmatrix}+[C_p(x)]\begin{bmatrix}\dot{u}(x,t)\\ \dot{v}(x,t)\end{bmatrix}+[K_p(x)]\begin{bmatrix}u(x,t)\\ v(x,t)\end{bmatrix}=\begin{bmatrix}p(x,0,t)+p_A(x,t)\\ 0\end{bmatrix}$$

$$p_A(x,t)=\gamma(u)\{c_4(x)[\dot{u}(x,t)-\dot{v}(x,t)]+k_4(x)[u(x,t)-v(x,t)]\}$$

$$\gamma(u(x,t))=\frac{\gamma_0(x)}{1+\mathcal{L}(u(x,t)^2)}$$

Mathematical Model



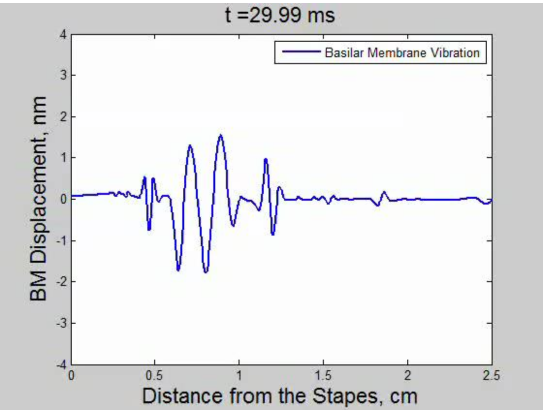
$$\mathcal{E}\dot{\psi}(x,t) = \mathcal{A}_u\psi(x,t) + \mathcal{B}\ddot{s}(t)$$

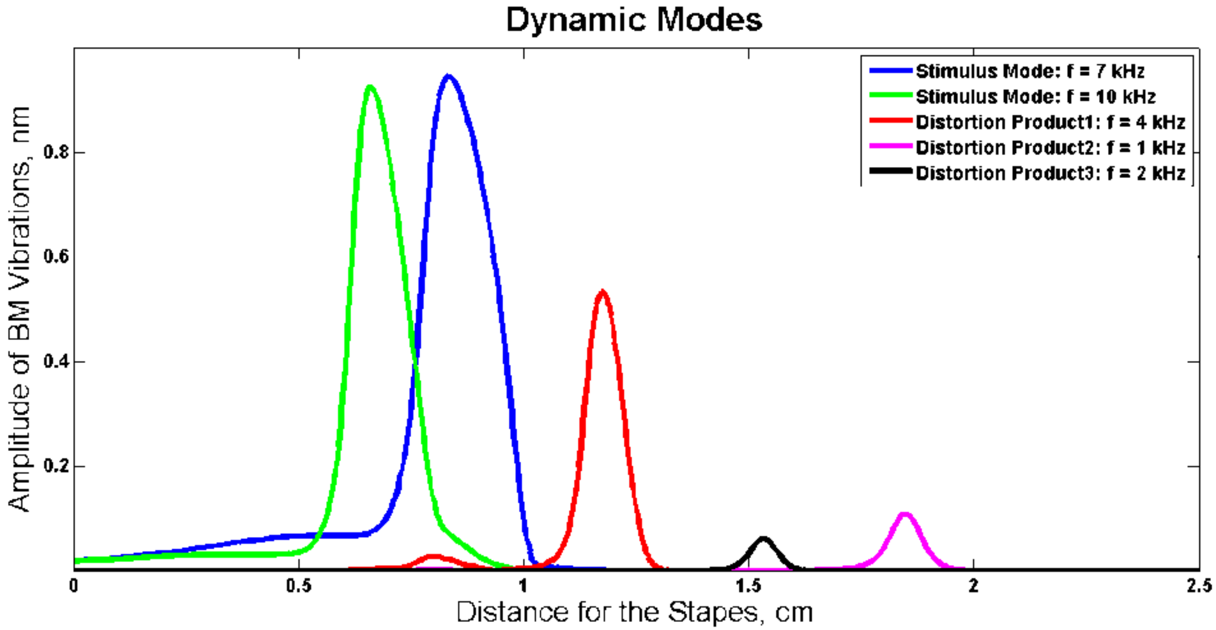
$$\psi(x,t) = \begin{bmatrix} u(x,t) \\ v(x,t) \\ \dot{u}(x,t) \\ \dot{v}(x,t) \end{bmatrix} \quad \mathcal{E} = \begin{bmatrix} \mathcal{I} & 0 & 0 & 0 \\ 0 & \mathcal{I} & 0 & 0 \\ 0 & 0 & m_1\mathcal{I} - \mathcal{L}_2 & 0 \\ 0 & 0 & 0 & m_2\mathcal{I} \end{bmatrix} \quad \mathcal{B} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{L}_1 \\ 0 \end{bmatrix}$$

$$\mathcal{A}_u = \begin{bmatrix} 0 & 0 & \mathcal{I} & 0 \\ 0 & 0 & 0 & \mathcal{I} \\ \gamma(u)k_4 - (k_1 + k_3) & -\gamma(u)k_4 + k_3 & \gamma(u)c_4 - (c_1 + c_3) & -\gamma(u)k_4 + k_3 \\ k_3 & -(k_2 + k_3) & c_3 & -(c_2 + c_3) \end{bmatrix}$$

Dynamic Mode Decomposition

Two Tone Sound Stimulus @ 7 kHz and 10 kHz





System Linearization

$$\gamma(u(x,t)) = \frac{\gamma_0(x)}{1 + \mathcal{L}(u(x,t)^2)}$$

$$\mathcal{E}\dot{\psi}(x,t) = \mathcal{A}_u\psi(x,t) + \mathcal{B}\ddot{s}(t)$$

$$\psi(x,t) = \begin{bmatrix} u(x,t) \\ v(x,t) \\ \dot{u}(x,t) \\ \dot{v}(x,t) \end{bmatrix} \quad \mathcal{E} = \begin{bmatrix} \mathcal{I} & 0 & 0 & 0 \\ 0 & \mathcal{I} & 0 & 0 \\ 0 & 0 & m_1\mathcal{I} - \mathcal{L}_2 & 0 \\ 0 & 0 & 0 & m_2\mathcal{I} \end{bmatrix} \quad \mathcal{B} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{L}_1 \\ 0 \end{bmatrix}$$

$$\mathcal{A}_u = \begin{bmatrix} 0 & 0 & \mathcal{I} & 0 \\ 0 & 0 & 0 & \mathcal{I} \\ \gamma(u)k_4 - (k_1 + k_3) & -\gamma(u)k_4 + k_3 & \gamma(u)c_4 - (c_1 + c_3) & -\gamma(u)k_4 + k_3 \\ k_3 & -(k_2 + k_3) & c_3 & -(c_2 + c_3) \end{bmatrix}$$

System Linearization

$$\gamma(u(x,t)) \longrightarrow \gamma_0(x)$$

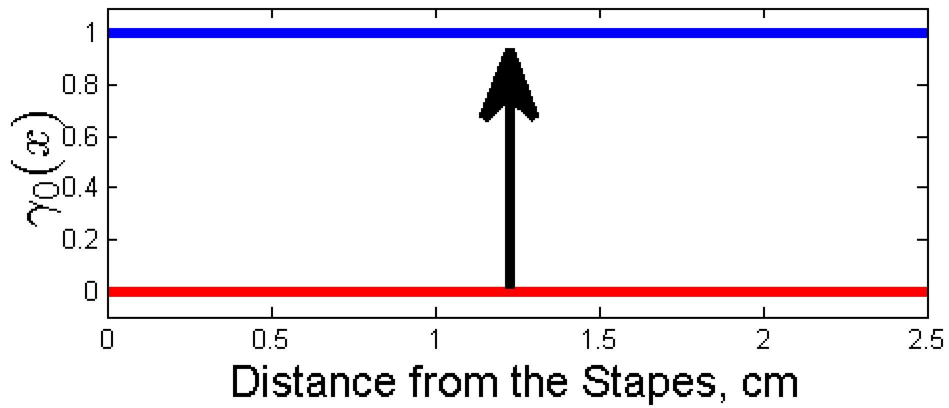
$$\mathcal{E}\dot{\psi}(x,t) = \mathcal{A}\psi(x,t) + \mathcal{B}\ddot{s}(t)$$

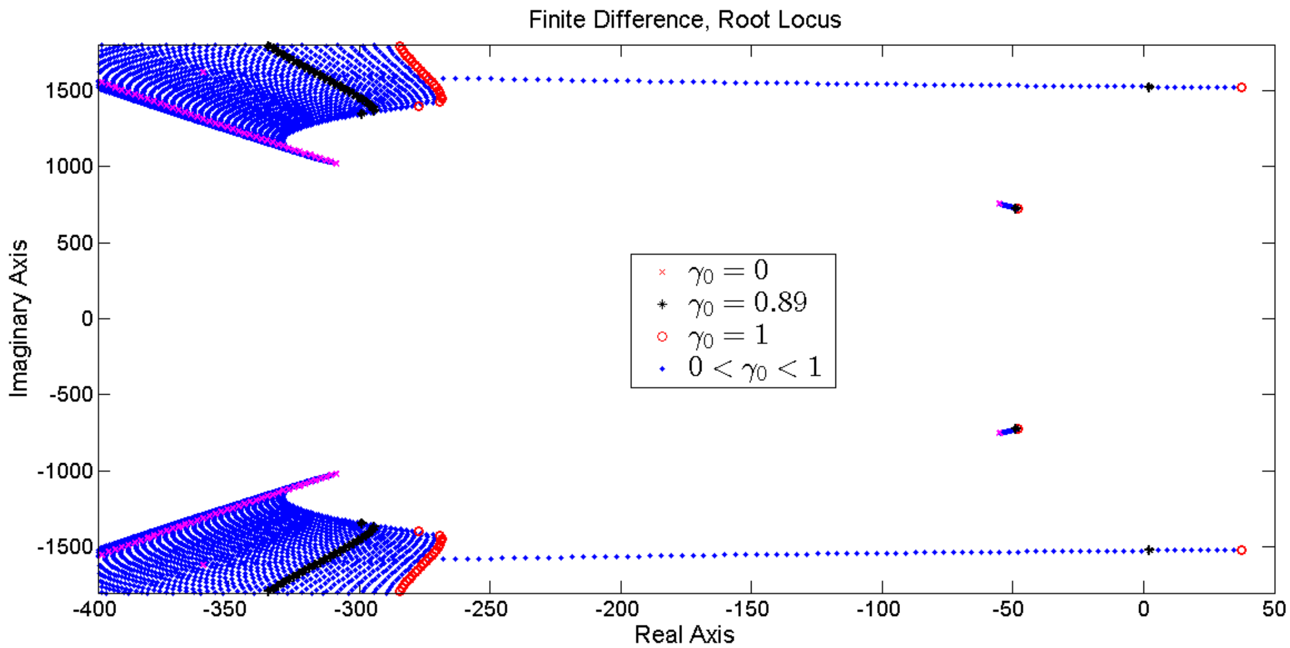
$$\psi(x,t) = \begin{bmatrix} u(x,t) \\ v(x,t) \\ \dot{u}(x,t) \\ \dot{v}(x,t) \end{bmatrix} \quad \mathcal{E} = \begin{bmatrix} \mathcal{I} & 0 & 0 & 0 \\ 0 & \mathcal{I} & 0 & 0 \\ 0 & 0 & m_1\mathcal{I} - \mathcal{L}_2 & 0 \\ 0 & 0 & 0 & m_2\mathcal{I} \end{bmatrix} \quad \mathcal{B} = \begin{bmatrix} 0 \\ 0 \\ \mathcal{L}_1 \\ 0 \end{bmatrix}$$

$$\mathcal{A} = \begin{bmatrix} 0 & 0 & \mathcal{I} & 0 \\ 0 & 0 & 0 & \mathcal{I} \\ \gamma_0(x)k_4 - (k_1 + k_3) & -\gamma_0(x)k_4 + k_3 & \gamma_0(x)c_4 - (c_1 + c_3) & -\gamma_0(x)k_4 + k_3 \\ k_3 & -(k_2 + k_3) & c_3 & -(c_2 + c_3) \end{bmatrix}$$

Stability Analysis (1)

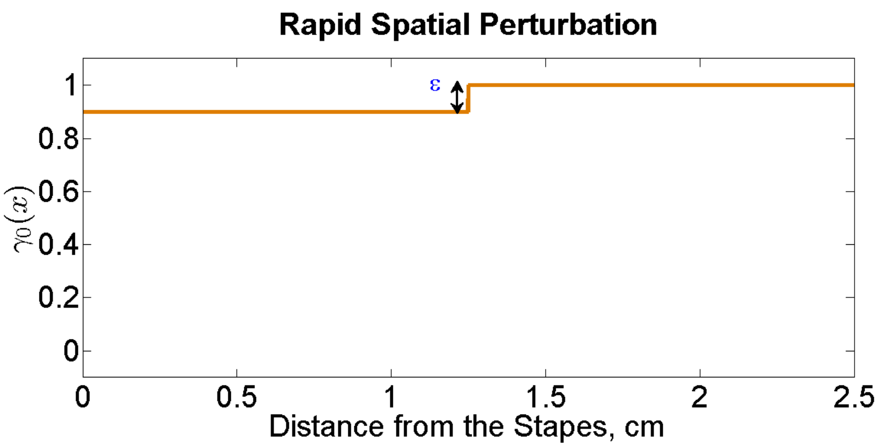
$\gamma_0(x) = \gamma_0 : 0 \rightarrow 1$





Stability Analysis (2)

$$\gamma_0(x) = 1 - \epsilon\gamma_1(x)$$



$$\dot{\psi}(x,t) = (\mathcal{E}^{-1}\mathcal{A})\psi(x,t) + (\mathcal{E}^{-1}\mathcal{B})\ddot{s}(t)$$

$$\mathcal{A} = \mathcal{A}_0 + \epsilon\mathcal{A}_1$$

$$\lambda = \lambda_0 + \epsilon\lambda_1 + \vartheta(\epsilon^2)$$

$$\lambda_1 = \frac{< \mathcal{E}^{-1}\mathcal{A}_1 v_o, w_0 >}{< v_o, w_0 >}$$

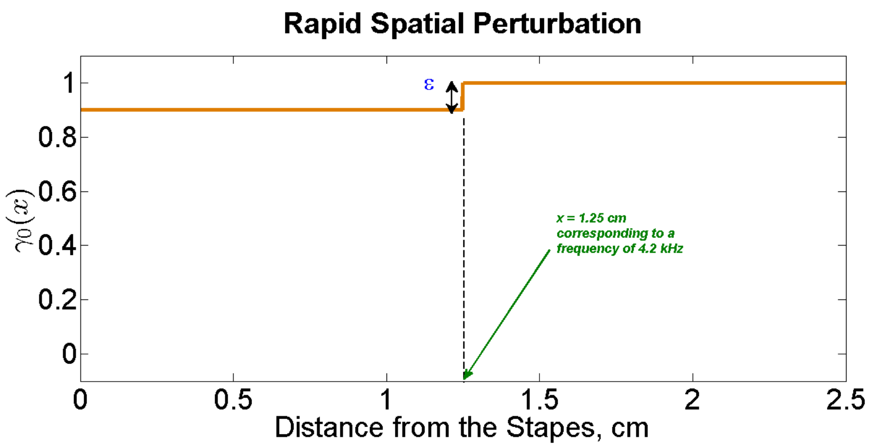
λ_0 : Eigenvalues of $\mathcal{E}^{-1}\mathcal{A}_0$

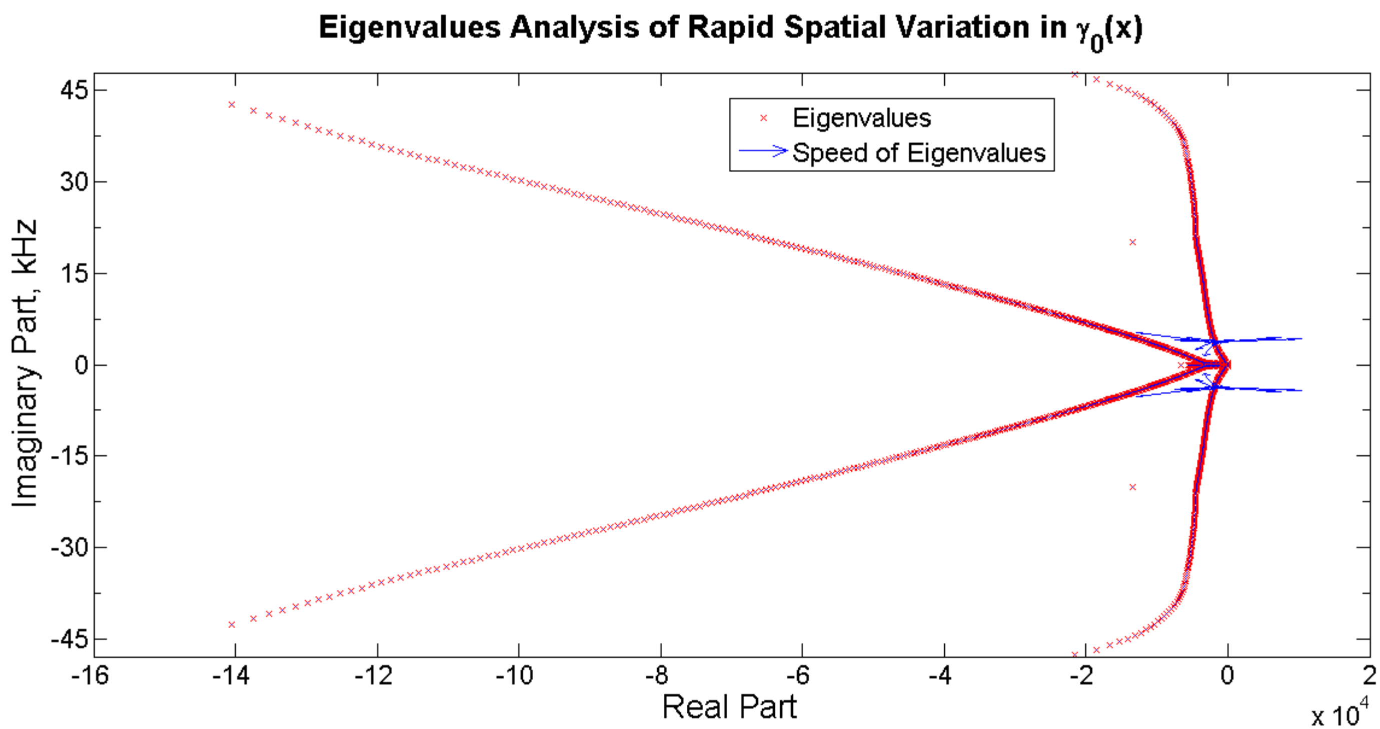
v_0 : Right Eigenvectors of $\mathcal{E}^{-1}\mathcal{A}_0$

w_0 : Left Eigenvectors of $\mathcal{E}^{-1}\mathcal{A}_0$

Stability Analysis (2)

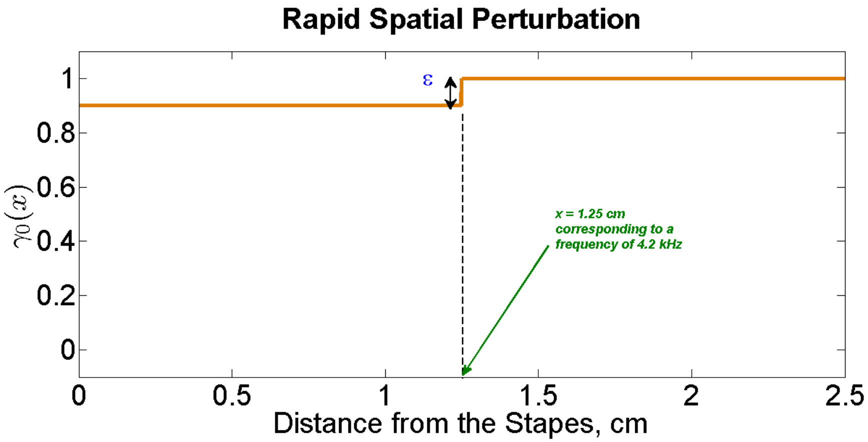
$$\gamma_0(x) = 1 - \epsilon\gamma_1(x)$$

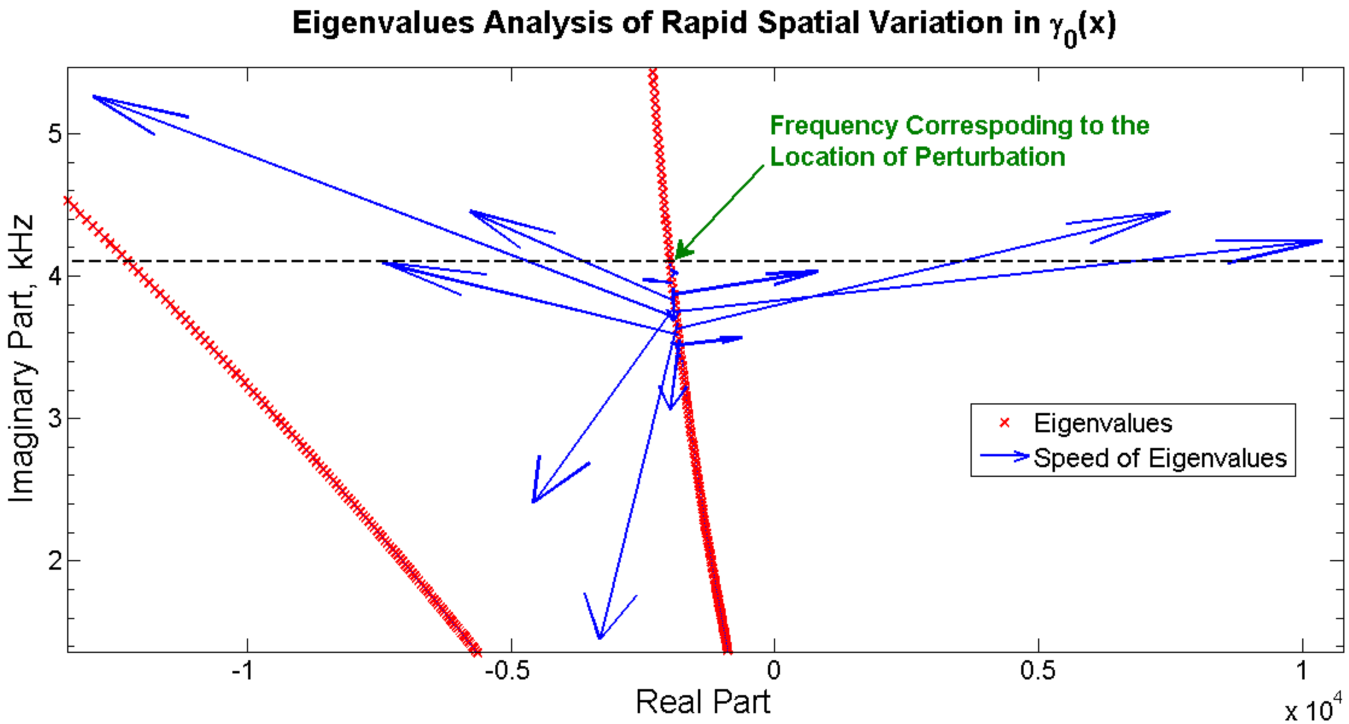




Stability Analysis (2)

$$\gamma_0(x) = 1 - \epsilon\gamma_1(x)$$





Conclusion

Possible Sources of Spontaneous Otoacoustic Emissions?

- High level of active gain

- Rapid spatial perturbations in the active gain